# Multi-Agent Dependence by Dependence Graphs

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## ABSTRACT
In this paper, we present an abstract structure called *dependence graph*, an extension of the notion of dependence network, as proposed in [16]. While this latter can be applied to express a set of dependence relations of a single agent, this new structure can be applied to the multi-agent case. It can be used, therefore, for the study of emerging social structures, such as groups and collectives, and may form a knowledge base for managing complexity in both competitive and organisational or other cooperative contexts. We analyze several properties of this structure, relating them to some corresponding social phenomena regarding group formation and cohesiveness.

## Categories and Subject Descriptors
I.2.11 [**Artificial Intelligence**]: Distributed Artificial Intelligence – *multiagent systems.*

## General Terms
Theory

## Keywords
Groups, teams, organizations and societies, group dynamics, formalisms for agents and MAS, self-organizing systems, emergent organization.

## 1. INTRODUCTION
This paper provides a formal model of multi-agent structures emerging from an aggregate of agents endowed with different goals and actions.

In [16], a notion of dependence network is proposed to represent the pattern of relationships holding between any given agent, on one hand, and one or more other agents on the other. In multi-agent systems, however, dependence relationships are *decentralised* structures, in which no agent involved is assigned a privileged role. As dependence networks are inadequate for representing decentralised structures, we propose instead a *graph* formalism to represent multi-agent dependence.

This new structure allows for the study of emerging social structures, such as groups and collectives, from simple aggregates of heterogeneous agents. Rather than situations in which tasks are already assigned and commitments made, we aim to model multi-agent interdependencies among different agents' goals and actions, and to build up a tool for predicting and simulating their emergence. This may form a knowledge base for managing multi-agent activity in both competitive and organisational or other cooperative contexts.

The paper is organized as follows. We discuss the importance of this work in the next section. In the third section, the original dependence theory proposed in [16] and some contributions [7] are summarized. In the fourth section, we present the formal description of dependence graphs. We use this notion in the fifth section, to illustrate how the dependence theory can be extended to include multi-agent dependence, with a special reference to group and collective phenomena. In the final section, some conclusions are drawn and ideas for future work are outlined.

## 2. MOTIVATION
The emergence and representation of social structures is a matter of growing concern in the (Multi) Agent Systems field [12] [13] [1]. This is so for different but interrelated reasons.

First, the emergence of groups, leadership and other social formations are receiving growing attention for designing and implementing robust open multi-agent systems [14]. Secondly, an efficient task distribution and execution is increasingly found to depend on dynamic, adaptive (self-) organised activity [2] [9] [8]. Thirdly, organisational practice is shown [4] to depend more on complex interrelationships among individuals and the environment rather than upon on an explicit hierarchical organisational design (cf. the PCANS model [11]). We are only too aware of the huge body of literature on theories, methods and techniques for exploring organisational structures, and for evaluating their performance (for a thorough analysis and a taxonomy, see [4]). Far from providing still another method or technique, we intend to propose a new perspective on the study of emerging organisational structure based upon systems of heterogeneous agents.

## 3. DEPENDENCE THEORY
The work presented in this paper proceeds from the assumption that heterogeneous agents endowed with goals, beliefs, able to perform actions and situated in a common world are involved in more or less complex and dynamic networks of relationships. In current agent systems, agents are often conceived of and designed as autonomous. However, they are not completely autonomous: agents may have goals that exceed or differ from their capacities to reach them. In particular, in teamwork, agents' autonomy is intrinsically limited [6].

More generally, socially situated agents may depend on one another to achieve their *own* goals. In terms of the dependence theory, an


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agent $ag_i$ depends on some other agent $ag_j$ with regard to one of its goal $g_k$, when:

1. $ag_i$ is not autonomous with regard to $g_k$: it lacks at least one of the actions or resources necessary to achieve $g_k$, while
2. $ag_j$ has the missing action/resource.

In the rest of this section, the dependence theory as presented in [16], on the basis of a pre-existing model developed by Castelfranchi et al. [5] is summarised. We also consider some extensions proposed in [7]. Only the notions relevant for the present exposition will be considered explicitly, namely those of *external description*, *dependence relations*, and *dependence networks*.

## 3.1 External Description

In order to be able to reason about the others, autonomous agents must have a data structure where the information about the others is stored, despite the possible different internal models they may have. This structure is called *external description*, and it is a private one, i.e., each agent has its own representation of the others. An external description is composed of several entries, each containing the beliefs that a certain *subject agent* has on a particular *object agent* that belongs to the agency.

An external description entry consists of the set of *goals* the object agent wants to achieve, the set of *actions* she is able to perform, the set of *resources* she controls and the set of *plans* she has. A plan consists of a sequence of actions with its associated resources needed to accomplish them. However, an agent may have a plan whose actions or resources do not necessarily belong to her own set of actions or resources. Therefore she may *depend on others* in order to carry on a certain plan, and achieve a certain goal.

As said before, the external description is an agency representation from within an agent's mind. One could ask what relationship holds between the agents' beliefs and the real matters. As one and the same instrument is intended to be employed for both types of representations (mental states and world states), in this paper an *objective representation* of the agency is adopted, namely that which corresponds to the mental state of a specific subject agent, assumed as the observer.

## 3.2 Dependence Relations

Let us suppose an agent $ag_i$ who tries to achieve a goal $g_k$. In the original formulation presented in [16] [15], the notions of autonomy and dependence are strictly related to the set of plans $P(ag_q, g_k)$ that the subject agent $ag_i$ uses in order to infer them[1]. For brevity, let us use respectively $p_{qk}$ and $P_{qk}$ as a shorthand notations for $p(ag_q, g_k)$ and $P(ag_q, g_k)$, with $p(ag_q, g_k) \in P(ag_q, g_k)$.

An agent $ag_i$ is *autonomous*[2] for a given goal $g_k$, according to a set of plans $P_{qk}$ if there is at least one plan $p_{qk}$ in this set that achieves this goal and every action $a_m$ appearing in this plan belongs to her own set of actions.

If an agent does not have all the actions to achieve a given goal, according to a set of plans, she may depend on others for this goal. An agent $ag_i$ *depends* on another agent $ag_j$ for a given goal $g_k$, according to a set of plans $P_{qk}$ if she has $g_k$ in her set of goals, she is not autonomous for $g_k$ and there is a plan $p_{qk}$ in $P_{qk}$ that achieves $g_k$ where at least one action used in this plan is in $ag_j$ 's set of actions.

An example of a basic dependence relation [15] could be:

$$dp_1\text{: } basic\_dep(ag_1, ag_2, g_1, p_{111} = a_1(),a_2(),a_4(), a_2)$$

which expresses that agent $ag_1$ depends on $ag_2$ to achieve goal $g_1$, because this latter may perform action $a_2$ needed in the plan $p_{111} = a_1(),a_2(),a_4()$ which achieves this goal.

Whenever two agents $ag_1$ and $ag_2$ depend on one another for their goals $g_1$ and $g_2$, there is a bilateral dependence relation between them. If their goals are the same ($g_1=g_2$), they have a *mutual dependence*; otherwise ($g_1 \neq g_2$), there is a *reciprocal dependence* between them.

## 3.3 Dependence Networks

Whenever an agent infers his basic dependence relations, he can internally represent them in a structure called *dependence network*.

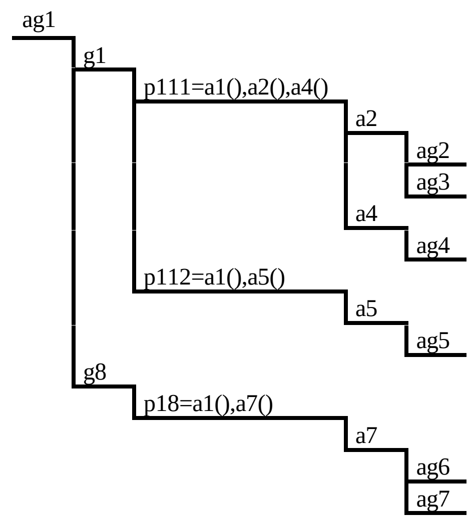


**Figure 1. Dependence Network**

As an example, let us consider the case where an agent $ag_1$ has two goals $g_1$ and $g_8$. For the first goal $g_1$, she has two alternative plans, $p_{111}= a_1(),a_2(),a_4()$ and $p_{112}= a_1(),a_5()$ to achieve this goal. On the other hand, for the second goal $g_8$, she has only one plan, $p_{18}= a_1(),a_7()$ that achieves this goal. Let us also suppose that $ag_1$ can perform actions $a_1$ and $a_9$, but she is unable to perform the actions $a_2$, $a_4$, $a_5$, and $a_7$, which can be performed respectively by the set of agents $\{ag_2, ag_3\}$, $ag_4$, $ag_5$ and by the set of agents $\{ag_6, ag_7\}$. In this scenario, the following basic dependence relations hold:

$$dp_1\text{: } basic\_dep(ag_1, ag_2, g_1, p_{111} = a_1(),a_2(),a_4(), a_2)$$
$$dp_2\text{: } basic\_dep(ag_1, ag_3, g_1, p_{111} = a_1(),a_2(),a_4(), a_2)$$
$$dp_3\text{: } basic\_dep(ag_1, ag_4, g_1, p_{111} = a_1(),a_2(),a_4(), a_4)$$
$$dp_4\text{: } basic\_dep(ag_1, ag_5, g_1, p_{112}= a_1(),a_5(), \quad a_5)$$
$$dp_5\text{: } basic\_dep(ag_1, ag_6, g_8, p_{18} = a_1(),a_7() \quad a_7)$$
$$dp_6\text{: } basic\_dep(ag_1, ag_7, g_8, p_{18} = a_1(),a_7(), \quad a_7)$$

The dependence network of $ag_1$ is presented in figure 1. In the general case, such a network can be much more complicated, as an agent can have several goals, with different plans to achieve them.

[1] The agent q whose plans are used to infer these notions is called source agent.

2 In the original theory, there were distinct definitions for action and resource autonomy and dependence. For simplicity only action-dependence will be considered, and we will call it dependence in the rest of the paper.

## 3.4 Some Extensions

One can notice that a basic dependence relation expresses a one-to-one relation. Some extensions were presented in [7], considering a one-to-many and many-to-one dependence relations. In the rest of the paper, $ag_i$ will denote a single agent, $Ag_i$ will denote a set of agents and $AG_i$ will denote a set of agents' sets.

### 3.4.1 OR-Dependence

An agent $ag_i$ OR-depends on a set of agents $Ag_j$ when she holds a disjunction set of dependence relations upon any member $ag_k$ of $Ag_j$. Any member of the set $Ag_j$ is sufficient but unnecessary for $ag_i$'s goal. For example, in order to have information about how to fill a tax form, any financial expert will do. OR-dependence provides the dependent agent with a number of alternative ways to achieve her goal, among which she shall choose the most convenient. The number of alternatives amounts to the number of agents contained in the set $Ag_j$. One can notice that *OR-dependence mitigates social dependence*, if only because the probabilities that some agent willing to help is found increase.

Referring to the dependence network presented in figure 1, the notion of OR-dependence is related to the fifth level of the network, i.e., *the possible agents able to perform a single needed action in a particular plan that achieves a certain goal*. In this network, $ag_1$ OR-depends on the set $Ag_2 =\{ag_2, ag_3\}$, because she needs one of them to perform action $a_2$ to achieve goal $g_1$, according to plan $p_{111}$.

### 3.4.2 AND-Dependence

Sometimes, one and the same agent may depend on a bunch of others for achieving one of her goals. For example, a rogue may AND-depend on a handful of specialised fellows to organise and execute a robbery: a lookout, a skilled driver, etc. If we consider that $ag_i$'s degree of dependence is a direct function of the costs of the actions that she needs to be performed, then, quite unlike the preceding link, *AND-dependence will be greater than ordinary dependence*, other things being equal.

Referring to the dependence network presented in figure 1, the notion of AND-dependence is related to the fourth level of the network, i.e., *the needed actions to perform a particular plan that achieves a certain goal*. Since different agents may perform a needed action, as was explained in the last subsection, the notion of AND-dependence must be built on OR-dependence.

In the network presented in figure 1, $ag_1$ AND-depends on the set of agents' sets $AG_3 =\{\{ag_2, ag_3\}, \{ag_4\}\}$, because he needs both actions $a_2$ (which can be performed either by $ag_2$ or by $ag_3$) and $a_4$ (which can be performed by $ag_4$) in order to execute $p_{111}$ to achieve goal $g_1$.

### 3.4.3 CO-Dependence

In this case, a set of agents $Ag_j$ depend on $ag_i$ each for its own goal. The lesser the actions that $ag_i$ can perform simultaneously, the more $ag_i$ will be contended for by $Ag_j$'s members.

# 4. DEPENDENCE GRAPHS

As one may observe, a dependence network contains all basic dependence relations of a *single agent*. Sometimes, it is useful to represent in a single structure several dependence networks, relating a *set of agents*. In order to do so, we introduce the notion of *dependence graphs*.

## 4.1 Definition

Mathematically, a graph $G$ is an ordered triple $(V(G), E(G), \psi_G)$ consisting of a nonempty set of $V(G)$ of vertices (or nodes), a set $E(G)$, disjoint from $V(G)$, of edges (or arcs) and an incidence function $\psi_G$ that associates with each edge of $G$ an unordered pair of (not necessarily distinct) vertices of $G$ [3]. The degree $d_G(v)$ of a vertex $v$ in $G$ is the number of edges incident with $v$, each loop counting as two edges. A graph $H$ is a subgraph of $G$ (written $H \subseteq G$) if $V(H) \subseteq V(G)$, $E(H) \subseteq E(G)$ and $\psi_H$ is the restriction of $\psi_G$ to $E(H)$. Let $v_0$ and $v_n$ be vertices in a graph. A *path* from $v_0$ to $v_n$ of length $m$ is an alternating sequence of $m+1$ vertices and $m$ edges beginning with vertex $v_0$ and ending with vertex $v_n$ $path(v_0, v_n)=(v_0, e_1, v_1, e_2, ..., e_m, v_n)$ in which edge $e_i$ is incident on vertices $v_{i-1}$ and $v_i$, *for* $i = 1,2,...,m$. A *simple path* from $v_i$ to $v_j$ is a path from $v_i$ to $v_j$ with no repeated vertices. A cycle (or circuit) $cycle(v_i)$ is a path of nonzero length from $v_i$ to $v_i$ with no repeated edges. A *simple cycle* is a cycle from $v_i$ to $v_i$ in which, except for the beginning and ending vertices that are both equal to $v_i$, there are no repeated vertices. A graph is *connected* if there is a path between any of its vertices.

A *tree* is a connected acyclic graph. A *rooted tree* $T(v)$ is a tree in which a particular vertex $v$ is designated the root. If $path(v_0, v_n)=(v_0, e_1, v_1, e_2, ..., e_m, v_n)$ is a simple path in a rooted tree $T(v_0)$, then $v_{i-1}$ is the *parent* of $v_i$ , $v_i$ is the *child* of $v_{i-1}$, and if $v_i$ has no children, $v_i$ is a terminal vertex (or leaf), for $i = 0, \ldots, n$. The *depth* of the root node is 0, and the depth of any node is the depth of its parent node plus one. The depth of the rooted tree $T(v)$ is the maximum depth of its nodes.

When the incidence function $\psi_g$ associates with the edge of $G$ an ordered pair of vertices, the graph is called a *directed graph* (or digraph). In this case, we can propose some new definitions for *directed paths* and *directed cycles*. If $G$ is a digraph and $\psi_G$ associates an edge $e_1$ of $G$ to an ordered pair of vertices $(v_0, v_1)$ of $G$, then $v_0$ is the *tail* of $e_1$, and $v_1$ is the *head* of $e_1$. The *indegree* $d_G^-(v)$ of a vertex $v$ in $G$ is the number of edges with head $v$, and the *outdegree* $d_G^+(v)$ of a vertex $v$ in $G$ is the number of edges with tail $v$.

A *bipartite graph* is one whose vertices can be partitioned into two subsets $X$ and $Y$, so that each edge has one end in $X$ and one end in $Y$. This definition can be extended to $n$ subsets, and the graph is called a *n-partite graph*.

Using these definitions, we can define a *dependence graph*. We will represent the basic notions of our model, agents, goals, plans and actions, as vertices of the graph. Moreover, as a pair of vertices will be linked by a single edge, we will represent a path in a simpler manner, without making an explicit reference to the edges, i.e., we will use $path(v_0, v_n)=(v_0, v_1, ..., v_n)$ instead of $(v_0, e_1, v_1, e_2, ..., e_m, v_n)$.

Formally, a dependence graph $DPG=(V(DPG), E(DPG), \psi_{DPG})$ is a 4-partite directed graph with the following characteristics:

1. the set $V(DPG)=V_{ag}(DPG) \cup V_g(DPG) \cup V_p(DPG) \cup V_a(DPG)$ is the union of the following disjoint sets:

    *1.1.* $V_{ag}(DPG)=\{ag_1, ag_2, ..., ag_n\}$ is the set of agents;

    *1.2* $V_g(DPG)=\{g_1, g_2, ..., g_n\}$ is the set of the possible goals these agents may want to achieve;

    *1.3.* $V_p(DPG)=\{p_1, p_2, ..., p_n\}$ is the set of plans the agents may use to achieve their goals;

    *1.4.* $V_a(DPG)=\{a_1, a_2 ..., a_n\}$ is the set of actions that can be performed by these agents.

2. the set $E(DPG)$ is a set of edges;

3. the function $\psi_{DPG}$: $E(DPG) \rightarrow V(DPG) \times V(DPG)$ is defined as follows:

*3.1.* $\psi_{DPG}(e) = (ag_i, g_j)$ associates an edge *e* with an ordered pair of vertices $(ag_i, g_j)$, with $ag_i \in V_{ag}(DPG)$ and $g_j \in V_g(DPG)$ and represents the fact that $ag_i$ has the goal $g_j$;

*3.2.* $\psi_{DPG}(e) = (g_i, p_j)$ associates an edge *e* with an ordered pair of vertices $(g_i, p_j)$, with $g_i \in V_g(DPG)$ and $p_j \in V_p(DPG)$ and represents the fact that goal $g_i$ *is* achieved by plan $p_j$;

*3.3.* $\psi_{DPG}(e) = (p_i, a_j)$ associates an edge *e* with an ordered pair of vertices $(p_i, a_j)$, with $p_i \in V_p(DPG)$ and $a_j \in V_a(DPG)$ and represents the fact that plan $p_i$ needs the action $a_j$ and agent $ag_k$ can not perform this action[3];

*3.4.* $\psi_{DPG}(e) = (a_i, ag_j)$ associates an edge *e* with an ordered pair of vertices $(a_i, ag_j)$, with $a_i \in V_a(DPG)$ and $ag_j \in V_{ag}(DPG)$ and represents the fact that action $a_i$ can be performed by agent $ag_j$.

## 4.2 A Single Agent Example

As an example, the dependence network shown in figure 1 is presented as a dependence graph in figure 2. In a single agent case, like shown in figure 2, the dependence graph results in a tree.

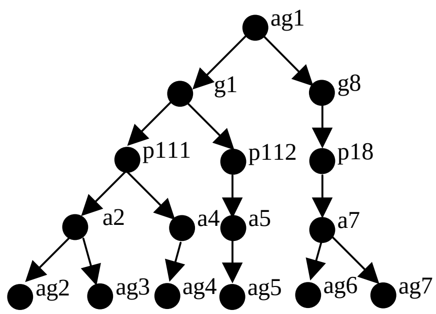


**Figure 2. Dependence Graph for a Single Agent**

One may notice the following interesting points:

1. a basic dependence relation *basic_dep*$(ag_i, ag_j, g_k, p_{qk}, a_m)$ is represented in the dependence graph by a simple path *path*$(ag_i,ag_j)=(ag_i, g_k, p_{qk}, a_m, ag_j)$ of length 4;

2. an OR-dependence of an agent $ag_i$ upon the agent set $Ag_j$ is represented in the dependence graph by a subtree $T(ag_i)$ of depth 4, containing one single vertex in each depth level 0, 1, 2 and 3 and at least two leaves in level 4, representing agents belonging to $Ag_j$;

3. an AND-dependence of an agent $ag_i$ upon the set of agents' sets $AG_j$ is represented in the dependence graph by a subtree $T(ag_i)$ of depth 4, containing one single vertex in each depth level 0, 1, and 2, and at least two vertices in level 3, representing different actions which can be performed by elements of $AG_j$.

## 4.3 A Multi-Agent Example

In the scenario presented in figure 2, the dependence graph represents only the goals of agent $ag_1$. However, as was stated in the introduction of this section, the interest of dependence graphs is that they can represent goals and dependence relations of *several* agents. In order to illustrate this concept, let us consider again the example shown in figures 1 and 2. Let us put forward the following additional hypothesis:

1. agent $ag_2$ has a goal $g_2$ and a certain plan $p_{22}= a_2(),a_6()$ to achieve this goal, she can perform action $a_2$ but not $a_6$. Another agent $ag_6$ can perform this last action, as well as action $a_7$, but she needs action $a_1$ to achieve her goal $g_6$, according to her plan $p_{66}= a_1(),a_6()$;

2. agent $ag_4$ has a goal $g_4$ and a certain plan $p_{44}= a_4(),a_7()$ to achieve this goal, she can perform action $a_4$ but not $a_7$. As well as $ag_6$, another agent $ag_7$ can perform this last action, but she needs action $a_9$ to achieve her goal $g_7$, according to her plan $p_{77}= a_7(),a_9()$;

In this scenario, the following additional basic dependence relations hold:

$$dp_7\text{: } basic\_dep(ag_2, ag_6, g_2, p_{22}= a_2(),a_6(), a_6)$$
$$dp_8\text{: } basic\_dep(ag_4, ag_6, g_4, p_{44}= a_4(),a_7(), a_7)$$
$$dp_9\text{: } basic\_dep(ag_4, ag_7, g_4, p_{44}= a_4(),a_7(), a_7)$$
$$dp_{10}\text{: } basic\_dep(ag_6, ag_1, g_6, p_{66}= a_1(),a_6(), a_1)$$
$$dp_{11}\text{: } basic\_dep(ag_7, ag_1, g_7, p_{77}= a_7(),a_9(), a_9)$$

The dependence graph for this scenario is shown in figure 3.

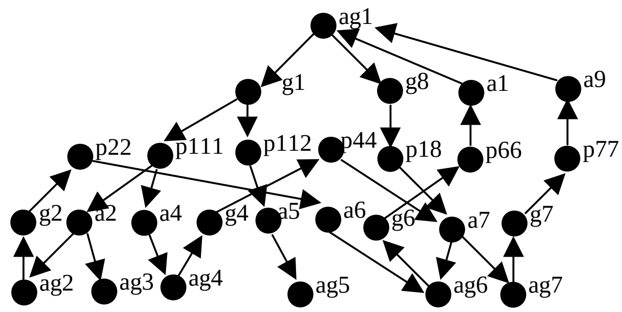


**Figure 3. A Multi-Agent Dependence Graph**

## 4.4 Reduced Dependence Graphs

As one may notice, the dependence graph is quite a complex structure with lots of paths. In some situations, like the ones that will be treated in the rest of the paper, this structure can be simplified.

Let us consider the case where (i) each agent has a *single goal* to achieve and (ii) each agent has a *single plan* to achieve this goal. In this case, we do not need to represent in the graph neither the vertices representing goals nor the ones representing plans, and the set of vertices will be reduced to the union of sets of agents and actions vertices $V(DPG) = V_{ag}(DPG) \cup V_a(DPG)$. Consequently, the function $\psi_{DPG}(e)$ will also be simplified, containing either elements like $\psi_{DPG}(e) = (ag_i, a_j)$ or $\psi_{DPG}(e) = (a_i, ag_j)$.

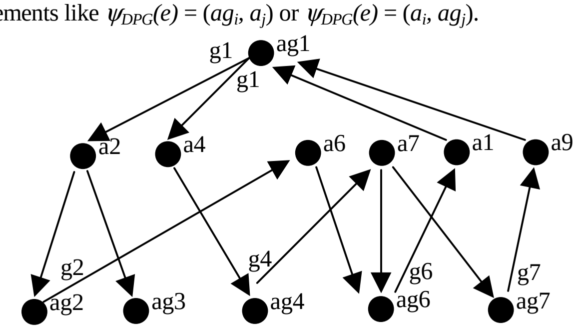


**Figure 4. Reduced Dependence Graph**

As it may be important for the further discussion to identify the agents' goals, we will use them as *labels* of the directed edges

[3] Agent $ag_k$ is the origin of the path to which this edge belongs, as illustrated in the sequence.

linking agents to actions. We will call this representation a *reduced dependence graph*. An example of such a graph is presented in figure 4, which corresponds to the scenario presented in the last subsection.

## 5. MULTI-AGENT DEPENDENCE

The above concepts and the whole set of edges in a dependence graph can be used to describe more or less structured and complex multi-agent systems. In particular, different levels of complexity and internal cohesiveness/fragility of a multi-agent system can be shown to emerge from some features of the dependence graph described above.

As will be shown throughout this section, rather than a none-or-all notion, multi-agent dependence indicates a phenomenon of growing complexity, from loose group-dependence to a more structured and more cohesive collective dependence.

In the remaining of the paper, for simplicity, we consider a set $Ag_i$ of non-autonomous agents with regard to their goals: each of them needs one or more actions, different from one another, to achieve their goals. Any member $ag_j$ of $Ag_i$ tries to achieve *a single goal*, and has *one single plan* that achieves this goal. Therefore, we will use reduced dependence graph to describe them.

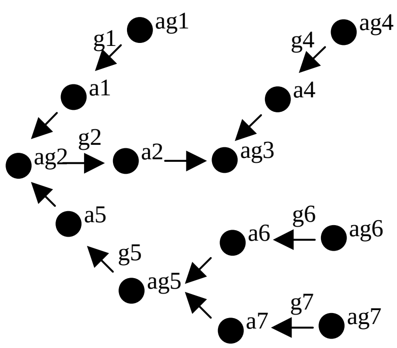


**Figure 5. An Acyclic Dependence Graph**

Figure 5 shown an acyclic dependence graph. This graph depicts a loose social relation: no partnership can emerge. Let us now see under which conditions the dependence graph may favour possible multi-agent partnerships.

### 5.1 AMONG-Dependence

Let us look now at figure 6. The graph contains two cycles, $cycle(ag_1)=\{ag_1, a_1, ag_2, a_2, ag_7, a_7, ag_1\}$ and $cycle(ag_4)=\{ag_4, a_4, ag_3, a_3, ag_4\}$ of different lengths: $cycle(ag_4)$ has length 4, while $cycle(ag_1)$ has length 6. The first one represents a not so complex situation: two agents depend on one another for their different goal (reciprocal dependence) [6]. Conversely, $cycle(ag_1)$ represents a more complex situation: it contains more than two agents, where each may receive help from someone and may provide help to another. Sociologists [17] would say that in $cycle(ag_1)$ a "generalised" form of exchange, requiring a rather complex negotiation process, might occur. We will call AMONG-dependence the dependence relationship holding in $cycle(ag_1)$.

More precisely, a set of agents $Ag_i$ can be said to AMONG-dependent when the following three clauses hold:

***Dependence clause*** - for each $ag_j$ belonging to $Ag_i$ there is at least another agent $ag_k$ upon which $ag_j$ depends for her goal;

***Utility clause*** - there is at least another agent $ag_l$ belonging to $Ag_i$ which depends upon $ag_j$ for his goal;

***Generalised reciprocity clause*** – $ag_k \neq ag_l$.

Considering the dependence graph, an AMONG-Dependence will hold if there is at least one subgraph $G \subseteq DPG$ which contains a cycle whose length is greater than 4.

Obviously, the larger the set of AMONG-depending agents, the more complex the structure and the more difficult the agreement and the coordination[4].

In figure 6, a simple and ideal case of AMONG-dependence is offered, represented by $cycle(ag_1)=\{ag_1, a_1, ag_2, a_2, ag_7, a_7, ag_1\}$. Under certain conditions, which we are going to examine, an AMONG-dependence may prove fragile, and either split or collapse.

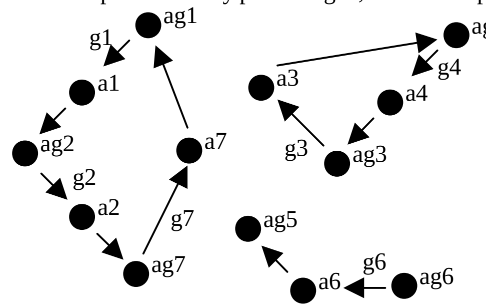


**Figure 6. AMONG-Dependence**

#### 5.1.1 AMONG-Dependence and OR-Dependence

Let us consider what happens if one agent $ag_j$, belonging to a set of agents $Ag_i$ which satisfies the AMONG-dependence clauses, OR-depends on a subset $Ag_k \subseteq Ag_i$, as shown in figure 7. Here, we have one agent ($ag_1$) OR-depending on two agents ($ag_2$ and $ag_6$) for action $a_1$. An interesting phenomenon emerges:

*If one agent $ag_j$, belonging to a set of agents $Ag_i$ which satisfies the AMONG-dependence clauses, OR-depends upon a subset $Ag_k$ of $Ag_i$, there are two or more cycles of different lengths starting at agent $ag_j$ which may satisfy the AMONG-dependence clauses.*

Considering the dependence graph *DPG*, in this situation there is at least one node $a_i \in V_a(DPG)$ ($a_1$ in figure 7) whose outdegree is greater that its indegree, i.e., with $d_G^+(a_i) > d_G^-(a_i)$.

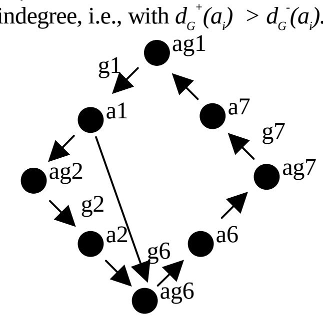


**Figure 7. Inequity in AMONG-Dependence**

When this property holds, there will be at least one agent that is more useful (more required) than dependent ($ag_6$ in figure 7). Consequently, the dependence graph will have two or more cycles, of different complexity.

[4]AMONG-dependence could be quantified as a function of the number of agents belonging to the set of agents involved (and the number of steps that should be realised in order for all agents to receive and give help). However, other variables should be taken into account as will be seen later on in the paper.

In figure 7, for example, these are the two cycles starting at vertex $ag_1$, $cycle_1(ag_1)=\{ag_1, a_1, ag_6, a_6, ag_7, a_7, ag_1\}$ and $cycle_2(ag_1)=\{ag_1, a_1, ag_2, a_2, ag_6, a_6, ag_7, a_7, ag_1\}$ and both satisfy the AMONG-dependence clauses. The OR-depending agent ($ag_1$) will have a chance to decide. Its choice will determine whether a sub-group will be formed at the expense of a (subset of) agent(s) (in our case, at $ag_2$'s expense). OR-dependence is an obstacle for AMONG-dependence to lead to group formation. Since OR-dependence reduces the degree of dependence, an OR-depending agent involved in an AMONG-depending network will always be less depending than useful. This introduces an unbalance, or inequity, in the network at the benefit of the OR-depending agent. As will be shown below, unbalance or inequity endangers AMONG-dependence and obstacles group formation.

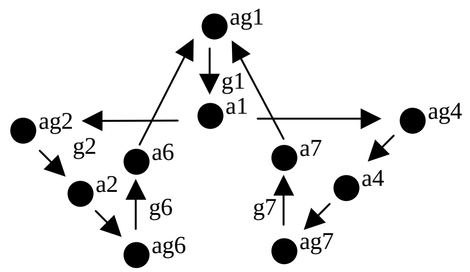


**Figure 8. Incompatibility in AMONG-Dependence**

A special case occurs when the OR-depending agent is useful to the same number of agents that she OR-depends upon, shown in figure 8:

*If the number of agents in the subset $Ag_l$ depending upon $ag_j$ is equal to the number of agents of the subset $Ag_k$ $ag_j$ OR-depend upon, there are alternative incompatible cycles starting at agent $ag_j$ in all of which AMONG-dependence clauses apply.*

Considering the dependence graph *DPG*, in this situation there is at least one node $a_j \in V_a(DPG)$ with $d_G^+(a_j) > d_G^-(a_j)$ ($a_1$ in figure 8), one node $ag_i \in V_{ag}(DPG)$ with $d_G^+(ag_i) < d_G^-(ag_i)$ ($ag_1$ in figure 8), and there is an edge $e \in E(DPG)$ with $\psi_{DPG}(e) = (ag_i, a_j)$.

In other words, an agent may OR-depend upon as many agents as those depending on her. In figure 8, for example, there are two incompatible cycles starting at vertex $ag_1$, $cycle_1(ag_1)=\{ag_1, a_1, ag_2, a_2, ag_6, a_6, ag_1\}$ and $cycle_2(ag_1)=\{ag_1, a_1, ag_4, a_4, ag_7, a_7, ag_1\}$, between which the OR-depending agent, contended for by the others, will have the power to choose. In either case, OR-dependence disrupts or obstacles AMONG-dependence and consequently, group formation.

### *5.1.2 AMONG-Dependence and AND-Dependence*

Let us consider what happens if we introduce an AND-dependence link, like the situation illustrated in figure 9.

In this case, group exchange can occur only if one of the agents accepts to give more than she receives (in figure 9, $ag_6$). Again, if at least one agent in the set depends more than it is useful for (inequity condition), the AMONG-dependence is seriously endangered and a fragile agreement is likely to emerge.

*If the set $Ag_i$ satisfies the AMONG-dependence conditions, but at least one agent $ag_j$ in $Ag_i$ AND-depends on a set of sets of agents $AG_k$ and $|AG_k| > |Ag_l|$, $Ag_l$ is the subset of agents depending on $ag_j$, AMONG-dependence is endangered.*

Considering the dependence graph *DPG*, in this situation there is at least one two nodes $ag_i, ag_j \in V_{ag}(DPG)$ ($ag_1$ and $ag_6$ in figure 9) with $d_G^+(ag_i) < d_G^-(ag_i)$ and $d_G^+(ag_j) > d_G^-(ag_j)$.

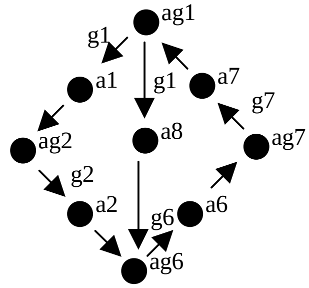


**Figure 9. Fragile AMONG-Dependence**

Nonetheless, sometimes AND-dependence may strengthen AMONG-dependence, as one may notice in figure 10. Here, once more, one can notice two cycles starting at vertex $ag_1$. However, they are not only compatible but actually inter-dependent: neither $cycle_1(ag_1)=\{ag_1, a_1, ag_2, a_2, ag_6, a_6, ag_1\}$ nor $cycle_2(ag_1)=\{ag_1, a_8, ag_4, a_4, ag_7, a_7, ag_1\}$ can independently form a group. They can only work together.

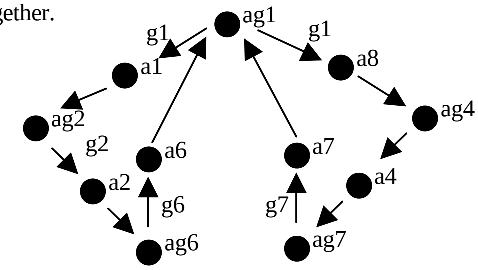


**Figure 10. Strong AMONG-Dependence**

*Given a set of agents $Ag_i$ which satisfies the AMONG-dependence clauses, if there is at least one agent $ag_j$ belonging to $Ag_i$ which AND-depends upon a set of sets of agents $AG_k$ and $|AG_k| = |Ag_l|$, $Ag_l$ is the subset of agents depending on $ag_j$, there are two or more compatible cycles, each satisfying the AMONG-dependence conditions.*

Considering the dependence graph *DPG*, in this situation for all nodes $ag_i \in V_{ag}(DPG)$ we have $d_G^+(ag_i) = d_G^-(ag_i)$. When this condition is not satisfied, the AMONG-depending graph is bound to either split or shrink, or group agreement is less likely to occur.

## 5.2 GROUP-Dependence

AMONG-dependence holding in a set of agents may give rise to GROUP-dependence when the equity clause is satisfied:

***Equity clause*** *- For all agents $ag_j$ belonging to the set $Ag_i$ of AMONG-depending agents, the subset of agents $Ag_k$ upon which $ag_j$ depends is equal to the subset $Ag_l$ of agents depending on $ag_j$.*

In this case, AMONG-dependence leads to a global network, a potential group structure, since no one can achieve its goal independent of the others' achievements and actions. This will be called *GROUP*-dependence.

Below, we will distinguish two conceptually different types of group dependence, which can co-exist in real matters.

### *5.2.1 Decentralized GROUP-Dependence*

Consider the following clause:

***Egalitarian clause*** *- For all agents $ag_j$ belonging to the set $Ag_i$ of AMONG-depending agents, $ag_j$ depends upon the same number of agents, and is useful for the same number of agents.*

When both the equity and the egalitarian clauses hold, a decentralised structure emerges: none plays a leading role, and all share an equal condition. All the agents involved may find an agreement leading them to receive what they need and provide what they are expected to deliver. If, and only if, all agents respect the agreement, the group-dependence will actually lead to an effective group partnership. Of course, a complex social process is necessary for the agreement both to be established and to be respected by all members. An example of a decentralised GROUP-dependence can be found in figure 6, if we consider exclusively the set of agents $Ag_i=\{ag_1, ag_2, ag_7\}$.

Considering the dependence graph *DPG*, in this situation for all nodes $ag_i \in V_{ag}(DPG)$ we have $d_G^+(ag_i) = d_G^-(ag_i) = n$.

### *5.2.2 Centralized GROUP-Dependence*

The fundamental dependence relationship that occurs in group-exchange is *reciprocal* dependence. Agents depend on one another each to achieve their own goals. Both reciprocal and group dependence, in fact, occur quite frequently in competitive contexts, like markets.

An intermediate phenomenon, bridging the gap between group and collective dependence, group exchange and teamwork, is centralised group dependence. Consider the dependence graph presented in figure 11. Here, one agent $ag_1$ AND-depends on all others to achieve her goal $g_1$, and all others depend on $ag_1$ to achieve each its own goal. There is a complete intersection among the set of agents $ag_1$ AND-depends upon and the set of agents depending upon $ag_1$.

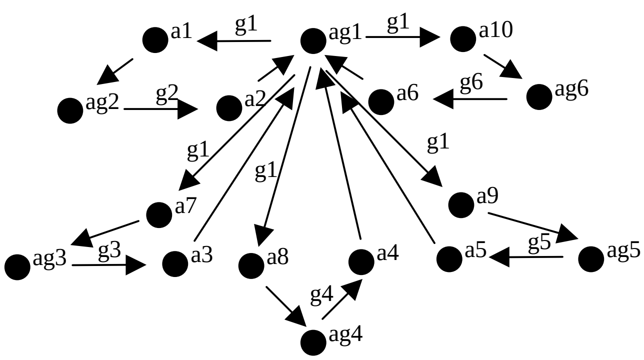


**Figure 11. Centralized GROUP-Dependence**

Centralised GROUP-dependence is a special case of AMONG-dependence in which the egalitarian clause does not apply (although the equity clause does), but there is at least one agent which plays a leading role. The dependence graph is centralised thanks to a multi-agent plan owned by one of the agents. All others are complementary for this one to execute her plan. This agent will "hire" all others to achieve this plan. In formal terms, centralised GROUP-dependence occurs in a set of AMONG-depending agents $Ag_i$ when all the following conditions hold:

1. the equity clause is satisfied;

2. there is at least one agent $ag_j$ belonging to $Ag_i$ which AND-depends on a set of sets of agents $AG_k$, where each element of $AG_k = \{\{ag_p\}, \{ag_q\}, \dots, \{ag_v\}\}$ is a singleton whose element $ag_x \in Ag_i$, $x = p, q, \dots v$ and $|AG_k| = |Ag_i|$-1;

3. any agent $ag_k$ belonging to $Ag_i$ depends upon $ag_j$.

In centralised group-dependence a group is likely to be formed and led by one agent which represents the head of the network.

Obviously, if one single agent OR-depends on all others, no group will emerge, but only two agents will form a partnership in exchange (the equity clause is not satisfied). Being the most useful and the least depending agent, the leader of the network would choose her partner for exchange.

## 5.3 COLLECTIVE-Dependence

Collective dependence is to group dependence what mutual dependence is to reciprocal dependence. The fundamental relationship of dependence is here *mutual* dependence [6]. Rather than a clear-cut distinction, however, group and collective dependence are situated on a continuum. AMONG-dependence graphs vary, among other factors (number of links, decentralised/non-decentralised), according to whether the agents involved share the goals with regard to which they depend on one another, or else pursue different goals. A COLLECTIVE-dependence holds in a set of AMONG-depending agents when each agent depends on all others to achieve a shared goal. This situation is represented in figure 12.

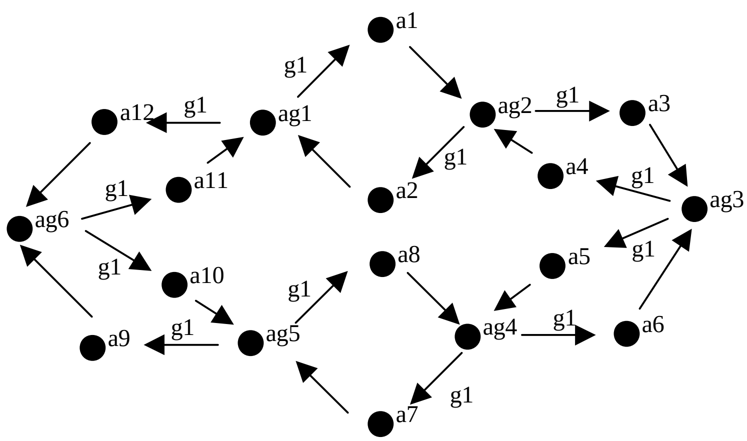


**Figure 12. COLLECTIVE-Dependence**

Collective dependence therefore leads to a rather cohesive group executing a multi-agent plan, i.e. teamwork. All agents are complementary to achieve one and the same goal.

In figure 12, a dependence graph where each agent depends on all others is shown. Such a network is inherent to the team and is based upon the multi-agent task that the team is supposed to accomplish. Full collective dependence occurs when the complementary agents share the goal for which they are needed.

In such a case, each member in the set of AMONG-depending agents depends on all others to achieve a common goal. A band of rogues offers a typical example of COLLECTIVE-dependence.

One may notice that COLLECTIVE-dependence need not be decentralised as in figure 12, if there is at least one agent $ag_j$ (or a subset of agents $Ag_j$) member of (or contained in) $Ag_i$ upon which all others depend upon in order to a leading role. A typical example is an orchestra, which is led by one member, the director.

## 6. CONCLUSIONS AND FURTHER WORK

In this paper, we have presented an abstract structure called dependence graph, which can be used to represent a set of dependence relations of a multi-agent system. A similar approach using graphs to represent cooperation structures is presented in [10], but our approach is richer, since in the former one could detect if a cooperation was possible to achieve a single agent goal.

To sum up, a bunch of agents searching partners for bargain are plunged in a more or less complex dependence graph, where groups and collectives arise. Rather than a clear-cut distinction, the difference between groups and collectives consist of different levels of complexity and cohesiveness of the underlying dependence graph. Full collective dependence occurs when the complementary agents share the goal for which they are needed. In such a case, each member in the set of complementary agents depends on all others to achieve the shared goal.

In the future, we intend to take the following steps: (i) to work out a formal model of the quantity of dependence; (ii) to incorporate the notions here presented in a computational system in order to check the efficiency of the model; and (iii) to run simulations to test its predictive power with regard to the emergence of groups and collectives.

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